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# Bow-shaped caustics from conical prisms: a 13th-century account of rainbow formation from Robert Grosseteste's *De iride*

JOSHUA S. HARVEY,<sup>1,2,\*</sup> HANNAH E. SMITHSON,<sup>2</sup> CLIVE R. SIVIOUR,<sup>1</sup> GILES E. M. GASPER,<sup>3</sup> SIGBJØRN O. SØNNESYN,<sup>3</sup> BRIAN K. TANNER,<sup>4</sup> AND TOM C. B. MCLEISH<sup>4</sup>

<sup>1</sup>Department of Engineering Science, University of Oxford, Oxford, UK

<sup>2</sup>Department of Experimental Psychology, University of Oxford, Oxford, UK

<sup>3</sup>Department of History, Durham University, Durham, UK

<sup>4</sup>Department of Physics, Durham University, Durham, UK

\*Corresponding author: [joshua.harvey@pmb.ox.ac.uk](mailto:joshua.harvey@pmb.ox.ac.uk)

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The rainbow has been the subject of discussion across a variety of historical periods and cultures, and numerous optical explanations have been suggested. Here, we further explore the scientific treatise *De iride* [On the Rainbow] written by Robert Grosseteste in the 13th century. Attempting to account for the shape of the rainbow, Grosseteste bases his explanation on the optical properties of transparent cones, which he claims can give rise to arc-shaped projections through refraction. By stating that atmospheric phenomena are reducible to the geometric optics of a conical prism, the *De iride* lays out a coherent and testable hypothesis. Through both physical experiment and physics-based simulation, we present a novel characterization of cone–light interactions, demonstrating that transparent cones do indeed give rise to bow-shaped caustics—a nonintuitive phenomenon that suggests Grosseteste’s theory of the rainbow is likely to have been grounded in observation.

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## 1. INTRODUCTION

In this paper we present an experimental approach to better understand the historical discussion of a natural optical phenomenon. Rainbows are occasionally observed meteorological phenomena that have stimulated mythic and aesthetic thought in many human cultures. These optical events have also provoked scientific discussion on the nature of light, color, geometry, clouds, and the atmosphere, leading to novel theory in both the physical and perceptual sciences.

The high medieval period saw advances in a diverse range of scientific fields, and perhaps chiefly that of theoretical and practical optics. A celebrated contributor to this advancement, Robert Grosseteste, explores the topic of the rainbow in his treatise *De iride* [On the Rainbow], written c.1228. Within the short text, one of 13 treatises written by Grosseteste on natural phenomena, an optical mechanism underpinning the rainbow is proposed. Previously, an interdisciplinary analysis has been applied to this treatise to explore its history (and likely dating), as well as its implications on color theory in the context of Grosseteste’s other writings [1]. Here, we apply a similar

research approach to the treatise’s focus on how the shape of a bow is made to appear in the sky, opposite the sun, to an observer. Central to this explanation is the notion of a transparent meteorological body in the shape of a cone, and the refractive events that occur when light passes through it. Although Grosseteste does not explicitly offer an exemplar observation of cone–light interaction, his theory is reliant on what he claims is a universal property of nature; when illuminated in a certain fashion, transparent cones collect light into bow-like shapes.

This claim, predicting what would be described in modern terminology as caustics, is not obvious and cannot be arrived at through logical deduction, intuition, or simple mathematics. The directional propagation of a single ray of light through transparent media can be described classically by the laws of internal and external reflection, and Snell’s law of refraction. If the refractive indices of media are known, and some trigonometric calculus is applied, the caustics of a sphere produced by parallel light may be modeled mathematically as the local maxima of distributed radiance. Apart from a sphere, whose

every plane-section is circular, the light paths through three-dimensional objects are complex to compute, or even visualize. A successful recent example of this has been the simulation of rainbows from physically accurate, nonspherical, raindrops [2]. But the geometric theory, optics, and mathematics required are far beyond the reach of a medieval scholar. Calculus was not formalized until the 17th century, and Willebrord Snellius was not born until the 16th century. While there is evidence that the law relating angles of refraction with angles of incidence was discovered earlier by the 10th-century Persian mathematician Ibn Sahl [3], it is clear that Grosseteste was unaware of it as he put forward his own, incorrect, law [4].

With these barriers in mind, we suggest that the claim Grosseteste makes about cone–light interaction, and that underpins his mechanistic explanation of the rainbow, was rooted in observation. While we would not describe this as experimental science, it would have required astute observation of the material world, and an appreciation of the uniformity of nature across vastly different contexts—from the human scale of a conical glass vessel at the dinner table to the meteorological scale of clouds and rainbows. Here we present a discussion of his theory, both its historical context and its content, and then explore his cone–light assertion further. The *De iride* fails to arrive at the correct explanation for rainbow formation, but along the way it expresses novel and coherent ideas. Through both physical experiment with water-filled transparent cones and physics-based rendering of caustics simulations, we show Grosseteste’s claim regarding cone–light interactions to be correct, and discuss its implications in the history of science.

## 2. EARLY HISTORY OF THE RAINBOW

The rainbow has been interpreted differently in a wide variety of human cultures. In Viking mythology it features as the bridge, Bifröst, between the realms of Gods and Humanity; for African and Australian peoples as the rainbow serpent—Kurreah for the Yulai of New South Wales; for Ancient Greeks, notably in the *Iliad*, as the messenger of the Gods [5]. Discussion of the rainbow in terms of both its colors and its shape occupied ancient and medieval authors considerably. To give an example of this and to situate Grosseteste’s thesis in its historical context, the interconnection and distinction between the two aspects can be seen by the presentation of rainbows in the Bible. The Latin translation of the Bible, made by St. Jerome in the later fourth century and the version that would have been known to Grosseteste, presents the rainbow shown by God to Noah as an arc in the sky with an intimate connection to clouds. The vision of heaven in the New Testament Apocalypse of St John (Revelations) refers, by contrast, to the rainbow by the Greek-derived word *iris*, the emphasis here placed on color.

The most influential explanation of the rainbow as a natural phenomenon in the ancient world was that of Aristotle in Book Three of his *Meteorology*, which presents a geometric, rather than a physical, argument for the appearance and shape of the rainbow [6]. For Aristotle, the rainbow is produced by reflection off a cloud and consists of three colors, and the shape of the bow is related to the shape of the clouds onto which it is projected [7]. The Aristotelian theory remained unchallenged, and

infrequently discussed in the West from Late Antiquity through to the High Middle Ages (c.1050–c.1250) [8]. Isidore of Seville attributes the shape of the bow to clouds and sun, and then identifies four colors related to the four elements. For Bede “rainbows come to be when the sun illuminates clouds.” Bede follows Isidore in his *De natura rerum* [On the Nature of Things], “The Rainbow with its four colors is formed in the air from the directly opposed sun and the clouds” [9].

By contrast, scholars working in Arabic in the early Middle Ages, notably Ibn Sīnā (Latinized as Avicenna) (980–1037) and Ibn al-Haytham (Alhazen) (c.965–1039), questioned the prevailing model, criticizing the identification of rainbows only with clouds, but in neither case making much advance on Aristotle’s position on the rainbow’s shape [10]. Alhazen’s optical theories would become influential in the West from the mid-13th century, affecting discussion of the rainbow by Roger Bacon, Witelo, and Theodoric of Freiberg [11].

Robert Grosseteste inherited a rather thin tradition of western scientific writing on the rainbow [12]. The new translations of Aristotle’s *Meteorology* and *Posterior Analytics* offered the most comprehensive account of the formation and appearance of the rainbow available to him [13,14]. It is these texts that form the basis of his own, fresh, engagement with the phenomenon and the existing authoritative sources on it. Less clear has been the extent by which he drew on his own observations, a question that we tackle from a new angle in this work.

## 3. ROBERT GROSSETESTE’S THEORY OF THE RAINBOW

Our forthcoming, related publication will present a new edition of and commentary on the *De iride*, and our present analysis of the text is based on the preparation of that edition and translation [15]. Grosseteste’s treatise consists of two distinct halves. The first concentrates on the medieval science of “perspective,” or optics. Grosseteste asserts that the study of the rainbow is subordinate to the study of optics—that is, that the study and understanding of a complex phenomenon should be broken down into the simpler study of its underlying component parts. For 13th-century natural philosophy, this reads with a modern-sounding scientific reductionism. Thus, the rainbow cannot simply be explained through its phenomenology, or observations alone, but requires an understanding of optical mechanisms.

Like many of his contemporaries, Grosseteste believed that sight is achieved by the emission of rays from the eye, though in the *De iride* he insisted on the idea of *synaugeia*—Plato’s proposition that rays from both the eye and the object were required in conjunction in order to facilitate sight [16]. The lengthy discussion on the physics of refraction, which we do not explore in this paper, includes a theorization of the law of refraction. Appealing to the simplicity of nature, as well as experimental evidence, Grosseteste states that the angle of refraction is equal to half the angle of incidence with respect to the normal. Earlier Islamic thinkers, of whose works Grosseteste was ignorant, had identified the correct law of refraction in the 10th century (Ibn Sahl) and, in the 11th, recorded Ptolemy’s second-century refraction experiment (Ibn al-Haytham) [10,17].

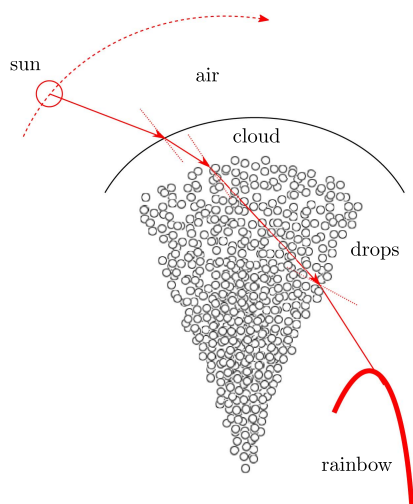
In consideration of the rainbow, Grosseteste frames his account exclusively as the refraction of rays coming from the sun, finding agreement with modern optics. Accordingly, the formation of the rainbow is explained by the propagation of light rays through several transparent (*diaphanus*) media and the refraction of the rays at the boundaries between them. In particular:

The sun rays first are refracted at the interface of air and the cloud, and thereafter at the interface of the cloud and the drops, so that by these refractions the rays converge in the density of the drops, and are there once more refracted and spread out as though from a pyramidal cone ... [15].

An interpretation of Grosseteste's theoretical meteorology and optical mechanism of rainbow formation is shown in Fig. 1, although it is important to state that no diagrams were drawn in the *De iride* and Grosseteste's descriptions are ambiguous, due to both the language employed and the limitations of translation from the original Latin. The curved top of a cloud with drops descending from it to the ground forms an inverted cone with a domed base. Light from the sun undergoes refraction at the air–cloud and cloud–drop interfaces, and subsequent refractive events among the drops themselves. Consequentially, light is gathered together within the cone. At the drop–air interface at the side of the cone, this gathered light is once again refracted, and spreads out:

In a shape similar to the curved surface of a round pyramid expanded opposite the sun. It therefore has the shape of an arch [15].

Although the meteorological scene is complex with several refractive events resulting in the gathering of radiance into a bow (which is then seen as a projection, perhaps onto some other cloud), Grosseteste states that the overall transformation of the incoming light is analogous to that produced by a simple transparent pyramidal cone. While this is far simpler to



**Fig. 1.** Interpretation of the geometric optics described in the *De iride*. Rays from the sun undergo three refractive events: the interface between the air and the dome of the cloud, within the cloud's drops, and upon re-entering the air. These events collect the radiance into the form of a bow. In this depiction a single ray from the sun is shown.

consider than a scene with numerous transparent media and associated refractive events at their boundaries, it is still a non-intuitive phenomenon as discussed earlier. Simply put, if asked what caustics are produced by a transparent cone when illuminated, most individuals will have no clear expectation, let alone base a theory of the formation of the rainbow on their conjecture. If transparent cones can produce such caustics, it is likely that Grosseteste knew this from observation.

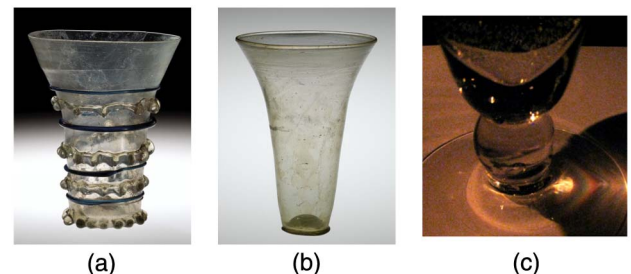
We tested Grosseteste's theory of cone-produced caustics with both physical reconstructions of cone–light interactions and physics-based simulations. Observations of cone–light interactions accessible to Grosseteste, which could have provided a scientific justification for this theory, would have featured either divergent or collimated light. It is possible that Grosseteste had observed bow-shaped caustics from cones in an indoor setting, such as a candlelit conical glass vessel, or in the open air, such as an icicle or glass vessel in sunshine.

#### 4. HISTORICAL RE-ENACTMENT OF CONE–LIGHT INTERACTION

We hypothesized that the most likely setting for Grosseteste to observe the caustics from an illuminated transparent cone was that of a water-filled glass vessel on a candlelit dining table. At least since the Roman empire, we have evidence of cone-shaped drinking vessels and beakers throughout early European and Middle Eastern history [18]. Two examples of conical beakers dated to the same period as Grosseteste are shown in Fig. 2, taken from the Web site of the Corning Museum of Glass [19].

To initially test our hypothesis, we filled a conical wine glass with water and illuminated it with a candle. The projected light collects into the shape of a bow, as depicted in Fig. 2. While this may not meet the modern definition of a caustic as the envelope of light rays transformed by refraction or reflection, there is a clear collecting of rays (in keeping with the origin of the term), resembling other optical phenomena that are colloquially referred to as caustics. The formation of this shape, and its striking display of colors, could well have provided an observation-based motivation for much of Grosseteste's *De iride*.

To more precisely investigate cone–light interactions, we sourced a transparent conical vessel and lighting with properties to mimic light both from the sun and from an indoor light



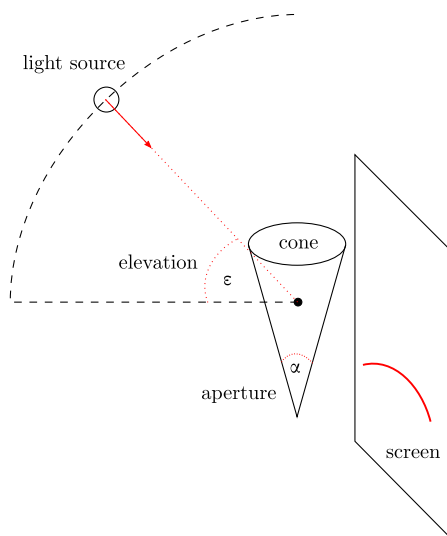
**Fig. 2.** Cone-shaped glass beakers dated to the same period as Grosseteste from (a) Central Europe and (b) the Middle East. (c) Modern conical wine glass filled with water, illuminated by a candle. A bow-shaped caustic can be seen to the right of the stem. The bow features a dispersive spectrum with the correctly ordered colors for a rainbow.



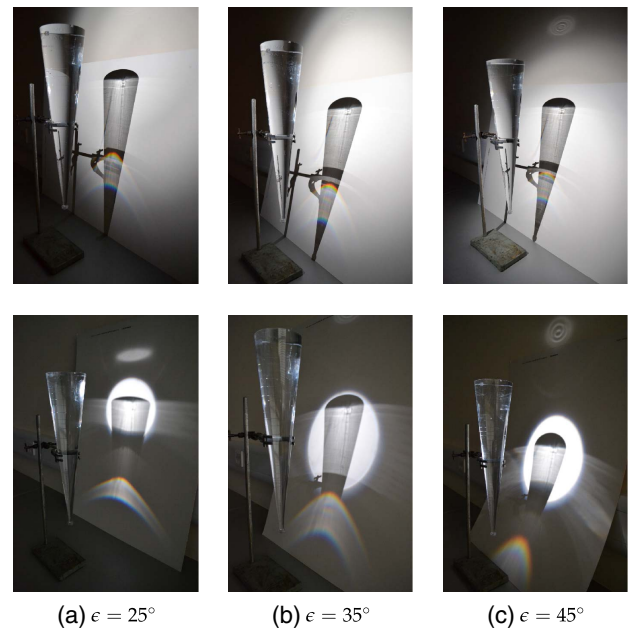
source such as a candle. A 1000 ml polycarbonate settling cone, with a base diameter of 110 mm, and an aperture (the angle formed in a cross section by the cone's lateral sides) of  $13.5^\circ$ , was completely filled with water and illuminated by a xenon lamp at a constant distance of approximately 400 mm from the cone, with a varying angle of elevation,  $\epsilon$ . The general plan of cone illumination is shown in Fig. 3, which also serves as the basis for later simulations when the cone aperture,  $\alpha$ , can be varied. A large white piece of card was placed approximately 150 mm behind the cone to serve as a screen for the projected caustics, and the setup was photographed. The light source was used both with and without a collimating lens, to recreate possible observations made by sunlight (either outside in the open air or inside using a *camera obscura*), or candlelight, respectively. Figure 4 shows that for a range of  $\epsilon$  values, a bow-shaped caustic is formed by both divergent and collimated illumination. Again, while this might not meet the definition of a caustic in the strictest sense (rays are not gathered into sharply defined lines or cusps), the projection appears similar to other optical phenomena, such as patterns of light seen within an illuminated swimming pool, that are described as caustics, and we adopt the term throughout the following discussion.

It was evident that the colorful bow-shaped caustics shown in Fig. 4 form from light entering the top of the cone, as small perturbations of the cone caused a rippling in the caustic. The bow was completely absent when the top of the cone was blocked with an opaque piece of plastic, as illustrated in Fig. 5. This is concordant with Grosseteste's theory of the rain-bow, where sunlight first enters the top of the cloud.

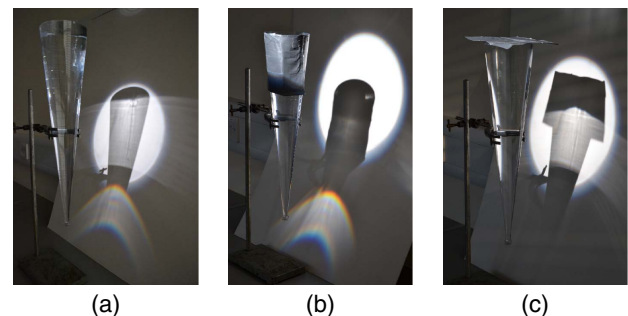
Although the focus of this paper is on the shape of caustics and the explanation in the *De iride* for the shape of the rainbow, it is worth considering here a number of additional characteristics of natural rainbows that are at least partially reproduced in these caustics. As can be seen in the photographs of Figs. 2 and 4, the bow-shaped caustics produced by transparent cones



**Fig. 3.** Parameters to describe the scene for both physical experiment and rendered simulations. The light source could emit either collimated light or divergent light.



**Fig. 4.** Physical experiment of cone-light interactions,  $\alpha = 13.5^\circ$ . Upper panel: illuminated by divergent light. Lower panel: illuminated using a collimating lens. Light elevation angles of (a)  $\epsilon = 25^\circ$ , (b)  $\epsilon = 35^\circ$ , and (c)  $\epsilon = 45^\circ$ .



**Fig. 5.** Light entering the top of the cone is responsible for the formation of the colorful bow seen projected onto the screen, collimated light,  $\epsilon = 35^\circ$ . (a) Unobstructed illumination, (b) blocking the side of the cone, and (c) blocking the top of the cone.

feature a strong display of colors, with red at the top of the bow as in a rainbow. Although Grosseteste is using cone-light interaction as an explanatory mechanism for the shape of the rainbow, it is highly likely that the conceptual leap from glassware to meteorology was mediated by the observation of color in both arcs. While these bows exhibit ordered variation in chromaticity, they are not the spectral colors that one sees from a beam of light exiting a dispersive prism. Rather, the colors appear desaturated to a varying degree, a feature of rainbows that has been discussed previously in connection to Grosseteste's theory of color [1,20]. Other features, such as Alexander's dark band, are also captured in the projection of many of these caustics, which may have contributed to the appeal of the cone-based theory.

## 5. PHYSICS-BASED RENDERING OF CONICAL PRISMS

To further explore cone–light interactions, physics-based rendering of simulated scenes was performed. As with the physical experiment, scenes were simulated to contain either an extended source of divergent light, such as a candle indoors, or a distant source of collimated light, such as sunlight in the open air or a *camera obscura*. Both scenes were constructed in the open-source program Blender to contain a light source, a camera, a transparent cone, and a screen onto which the cone’s shadow and any resultant caustics would be projected [21]. For scene descriptions, Blender-specific variables are printed in monospace. A degree of volumetric scattering was included in the Exterior Volume of the scene, so that rays could be visualized as they bunch together to form caustics. Further details for each scene can be found in the legend of Fig. 6, which contains the corresponding rendered images. Although many rendering techniques do not readily yield caustics, simulations have previously been used to explore the lighting and caustics that would result from early Islamic glass lamps [22]. For the work presented here, scenes were rendered with the Luxrender engine, a physics-based unbiased renderer that can be used to produce realistic images of caustics and other optical phenomena. While the study of the rainbow might naturally suggest an investigation into color and dispersive phenomena, scenes were initially rendered without dispersion as we were testing Grosseteste’s claims of how the shape of the arc is produced.

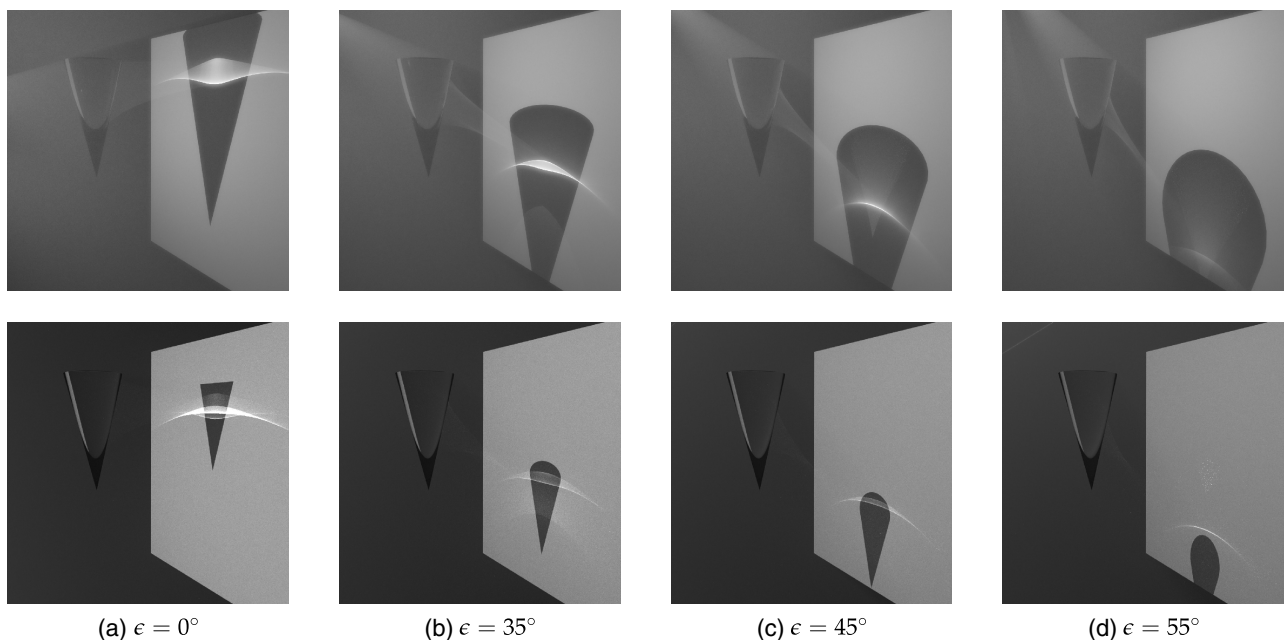
Cones with an aperture of  $27^\circ$  were constructed as Homogeneous Volumes, having a small amount of internal scattering, with a Glass surface, featuring a refractive index of 1.33 to model the optical properties of water. An opaque plane served as a screen for the projected shadow and caustics of the cone, positioned parallel to the cone’s axis. Simulations

were run with cones made with both flat and domed bases. While Grosseteste’s meteorological description features a cone with a domed base, the outwardly concave cloud, it is more likely that he observed caustics from cones with flat bases. Whether the bases of cones were flat or rounded did not have a strong impact on the resultant caustics, so for simplicity we have only presented the simulations featuring flat bases.

The first simulation used a small plane as an Area Light, emitting a spread of light over the scene from an extended source of light, to model a candle flame. The light object was positioned with locations and rotations corresponding to the desired ray angles and elevations. The second model simulated cone–light interaction in the open air by using the Luxrender Laser light source, which flooded the scene with collimated light at a designated angle. It therefore modeled which optical phenomena would be observed from a transparent cone in collimated light, such as an icicle in sunlight.

## 6. RENDERING METHOD

For rendering, Luxrender’s Metropolis Sampler was used in conjunction with the Bidirectional Rendering Mode. By using the Metropolis light transport algorithm to generate samples, and the bidirectional path tracer, the rate of convergence for rendering scenes with complex light–media interactions is greatly improved [23]. This improved efficiency toward convergence is especially valuable for scenes featuring caustics. Rendering parameters were adjusted to improve the detail of caustics and reduce image noise, and the Single Volume Integrator was used in conjunction with Noise-Aware Sampling. Renders were tonemapped using the Reinhard kernel to preserve details in highly illuminated regions of the image [24]. Rendering was halted when 99.99% of pixels passed a



**Fig. 6.** Images of caustics resulting from cone–light interactions simulated using the Luxrender engine in Blender. Upper panel: a simulation of the physical experiment photographed in Fig. 4, featuring an extended source of divergent light. Lower panel: a scene lit by collimated light to simulate an observation in the open air illuminated by the sun.

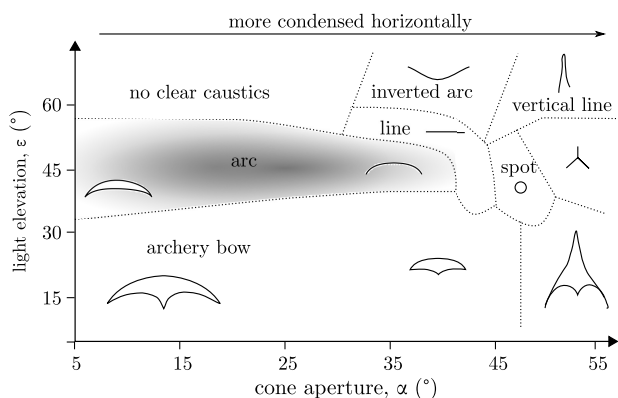
convergence test, producing images with a resolution of 1080 by 1080 pixels.

We note that a bidirectional path tracer, where samples are constructed through a combination of rays originating from the camera and rays from light sources, is essentially an implementation of Plato's *synaueia* theory of vision, advocated by Grosseteste in the *De iride*, discussed earlier.

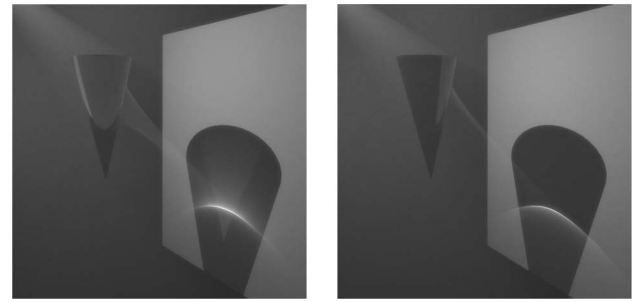
## 7. RESULTS AND DISCUSSION

As illustrated by Fig. 6, bow-shaped caustics are produced by both divergent and collimated light being projected through transparent cones for certain scene configurations. Luxrender simulations allowed for the construction of scenes with precise and modifiable geometry, such that a parameter space of cone–light interaction could be comprehensively explored. Two parameters were varied to investigate the refractive optical interactions with transparent cones: cone aperture (the angle formed in a cross section by the cone's lateral sides) and light source elevation (i.e., height and angle relative to cone, with a constant distance from the origin), as depicted in Fig. 3. Narrow cones gave rise to arc-like caustics across a broad range of light elevations (for  $\epsilon \approx 30 - 60^\circ$ ), but as aperture increased to a more martini-glass shape, arcs were less likely to form ( $\epsilon \approx 35 - 45^\circ$ ). For Luxrender simulations modeled with divergent light, we arrive at the parameter space illustrated in Fig. 7.

Not indicated in this parameter space is the proportion of light entering the top of the cone to that entering the side of the cone, although it logically follows that at low values of  $\alpha$  and  $\epsilon$ , light is predominantly passing through only the side. As values of  $\alpha$  and  $\epsilon$  increase, an increasing proportion of light enters the top of the cone, which, according to physical experiment and the mechanism described in the *De iride*, is the light responsible for the formation of bow-shaped caustics. To test whether this was also the case for the bow-shaped light projections seen in the simulations, a scene was rendered containing a cone with its front face constructed with an opaque mesh. As seen in Fig. 8, it is also the case for this simulation that bow-shaped caustics arise from light entering the top of the cone.



**Fig. 7.** Parameter space of cone-produced caustics obtained from Luxrender simulations, for scenes containing a divergent light source. The region where arcs are produced is shaded gray for emphasis.



**Fig. 8.** Light entering the top of the cone is collected into the bow-shaped caustic seen in the simulations. (a) Divergent light of  $\epsilon = 45^\circ$ . (b) The same scene, but the portion of the cone's side facing the light source is opaque.

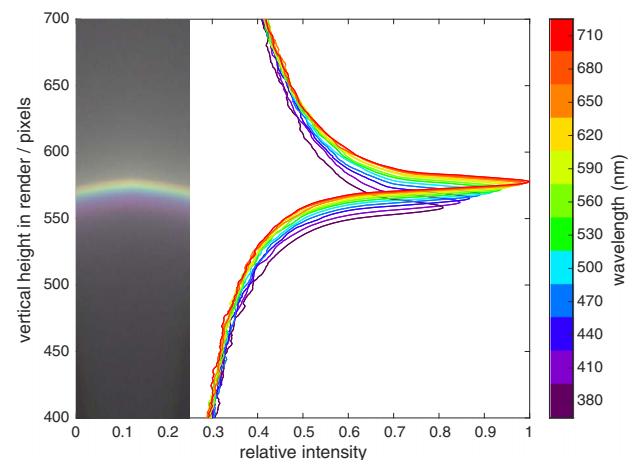
To simulate dispersive effects using Luxrender, 12 scenes were rendered containing cones of varying refractive indices, which corresponded to the effective refractive index values for light at wavelengths equally spaced between 380 and 710 nm, as calculated for water by the three-term Selmeier equation:

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3},$$

where  $n$  is the refractive index,  $\lambda$  is the wavelength, and  $B$  and  $C$  are experimentally determined Sellmeier coefficients [25].

The pixel intensities for each image were then plotted as a function of height through the render, as shown in Fig. 9, which includes a composite image of the 12 renders recolored according to wavelength. The caustic produced by the cone in the scene is dispersive, with light scattering to varying parts of the image as a function of wavelength.

During the physical experiment with the polycarbonate cone, another prominent bow-shaped caustic was observed to form at the same side of the cone that was illuminated,



**Fig. 9.** Simulated dispersive effects, after rendering images for 12 different wavelengths of light,  $\epsilon = 45^\circ$ ,  $\alpha = 27^\circ$ . On the left a composite image of 12 renders for light of wavelengths between 380 and 710 nm is shown. The plot shows the distribution of pixel intensities for each wavelength through the center of the rendered image.



as seen in Fig. 10. This presumably forms in the same way that transparent spheres (and, in particular, raindrops forming a rainbow) produce caustics, namely by an internal reflection off the back face. However, Luxrender was unable to generate this caustic. To further explore this, the MATLAB library Optometrika was used to construct a ray-tracing scene similar to that of the physical experiment [26]. An aspherical lens was defined to approximate a transparent cone, and screens were placed both below the cone and behind, to capture any projected rays that had or had not been internally reflected, respectively. The more precise and customizable nature of the ray-tracing calculated by Optometrika provided more insight into how caustics may be created by cones. Light entering the side of the cone undergoes a minor transformation upon exiting the other side, with some bunching together horizontally but remaining parallel in the vertical axis. This is to be expected, as rays hitting the cone at the same radial angle, but different heights, are self-similar. However, rays entering the top of the cone (and therefore in agreement with Grosseteste's optical mechanism) do form strong caustics.

As shown in Fig. 10, the internally reflected caustic is well modeled by Optometrika, and the radiance of the scene forms a tight bow in front of the base of the cone. However, no rays seem able to make it through the back face of the cone, and the original caustic observed in both physical experiment and Luxrender simulation is absent. More fundamentally, it is necessary to address the clear differences between the results of the physical experiment shown in Fig. 4, with the results from simulations. While both the Luxrender and Optometrika simulations can accurately model important features of cone–light interactions, neither could produce the full range of caustics observed with the physical cone. The shortcomings of these simulations reflect the limitations of Grosseteste's own theory,

which, while correctly describing some of the light projections from transparent cones, is silent regarding others.

## 8. CONCLUSION

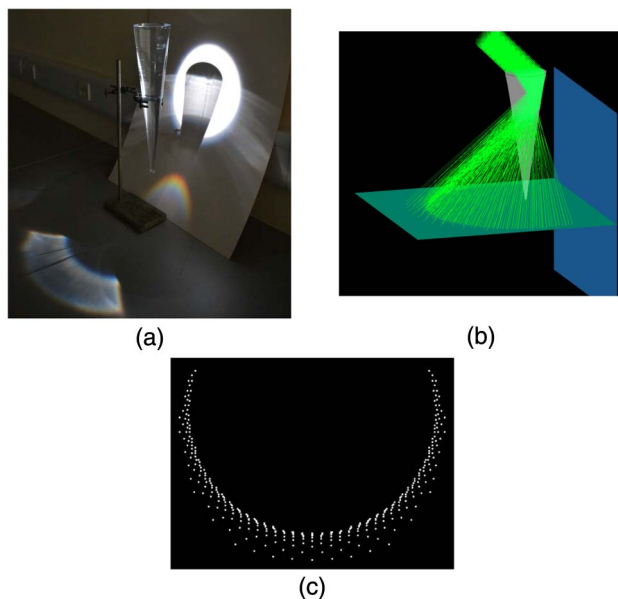
While the history of scientific discussion on the rainbow has been written about extensively, we are still in the process of discovering and rediscovering contributions that were influential in the development of the field. Grosseteste has not normally been given prominence in the development of scientific understanding of the rainbow. In *Theories of Vision*, Lindberg dismissively writes:

*Much has been made of Grosseteste's utilisation of refraction in explaining the rainbow, but the luster of that achievement quickly fades when we realise that his theory of the rainbow could not account for even the most basic phenomena and has remained largely unintelligible to the modern day [16].*

With this paper we have shown that, far from a lackluster and unintelligible theory, Grosseteste's *De iride* presents a novel, coherent, and pioneering theory of rainbow formation. By engaging with the text in an interdisciplinary way, we have uncovered features and implications that have been overlooked in previous discussions, and have found new evidence in support of Grosseteste's theory being grounded in observation of the material world. If this were the case, the *De iride* contains possibly the first attempt at scientific explanation of the rainbow motivated by material world observation, situating Grosseteste among other pioneers such as Aristotle, Ibn al-Haytham, Kamal al-Din al-Farisi, Theodor of Freiberg, and Descartes.

It is also interesting that while Grosseteste's theoretical mechanism of cone–light interaction can give rise to bow-shaped caustics as he claims, this is not universally applicable for all cones and all light angles. Rather, caustics form in the shape of a bow within a specific region of the parameter space mapped by varying cone aperture and light elevation. This too is somewhat addressed in the *De iride*, where Grosseteste explores theoretically the relationship between the elevation of the sun and the extent to which a rainbow forms. It therefore captures something of the rarity of rainbows, which form in nature only when particular conditions are met.

While the aim of this work has been to explore Grosseteste's theoretical explanation for the shaping of the rainbow, doing so has revealed deep-running internal consistencies in his writings, particularly between the *De iride* and his unique theory of color, stated in his treatise titled *De colore* [On Color]. In this treatise, Grosseteste describes three pairs of opposing qualities, which together form a three-dimensional space containing all possible colors [27]. A difficulty in understanding the text, and analyzing Grosseteste's color theory in general, has been the precise translation of the words he uses for these qualities. One of these pairs, describing a quality of the light, is identified as *multa-pauca*. Grosseteste asserts in the *De colore* that light is *multa* when it has been gathered together, such as by a burning glass. To a modern reader, in isolation of his other writings, this may seem to correlate simply with the intensity of light, and the question arises of how the gathering together of light might result in chromatic differences. Within the *De iride*, Grosseteste states that variations along the *multa-pauca* axis can account



**Fig. 10.** (a) Photograph of physical experiment and (b) Optometrika ray-tracing showing the bow-shaped caustic formed by internal reflection off the back face of the cone. (c) Rays projected onto the horizontal screen in the Optometrika simulation.  $\varepsilon = 45^\circ$ .



for the variations in color seen within a single rainbow. If, as we suggest, Grosseteste had observed variations in color arising from the gathering of light by transparent objects, this relationship between the term *multa* and variation in chromaticity would therefore seem to be an internal consistency in his understanding of color.

Grosseteste's theory of rainbow formation is not correct, and does not advance our understanding of the rainbow as a natural phenomenon today. However, studying it reveals a movement toward a natural philosophy based on careful observation of the material world, the theoretical use of similarity in physics between different scales, and testable prediction. It would not be long until this emerging experimentalist approach discovered the correct mechanism of rainbow formation.

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